ORTHOGONALIZATION WITH A NON-STANDARD INNER PRODUCT AND APPROXIMATE INVERSE PRECONDITIONING

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Staré Splavy

Conjugate gradient method

Assume linear algebraic system:

$$Ax = b,$$

where A is a SPD matrix.

Well-known fact: Eigenvalues determine convergence rate of CG method $\rightarrow$ preconditioned conjugate gradient method.

Preconditioners for iterative solvers:

- are necessary for increasing their robustness
- accelerate their convergence (by improving spectral properties of the matrix A)

Orthogonalization inner products

Gramm-Schmidt:

$$z_i^{(j)} = z_i^{(j-1)} - \alpha_{ji} z_j$$

$$z_i = z_i^{(j)} / \|z_i^{i-1}\|_A$$

Modified Gram-Schmidt:

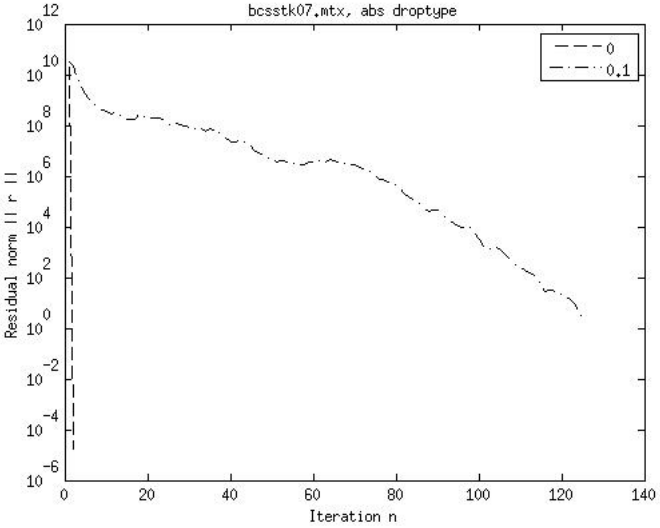
$$\alpha_{ji} = \langle z_i^{(j-1)}, z_j \rangle_A$$

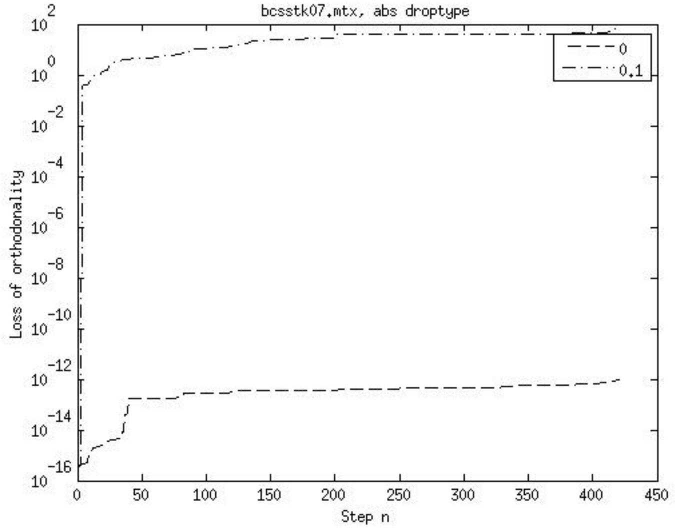
Classical Gram-Schmidt:

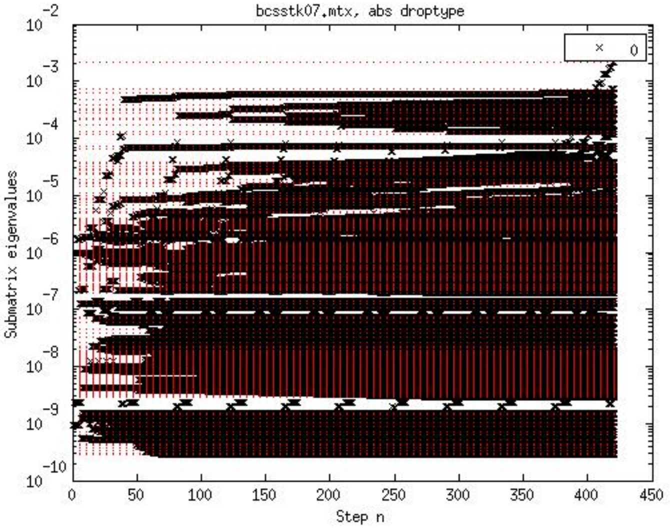
$$\alpha_{ji} = \langle z_i^{(0)}, z_j \rangle_A$$

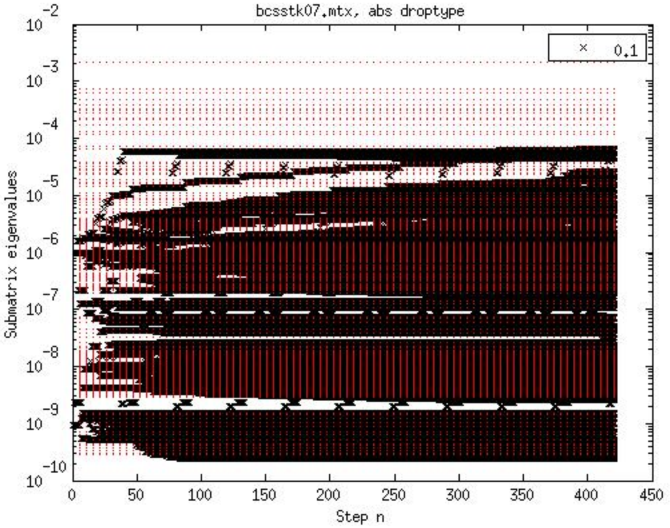
AINV:

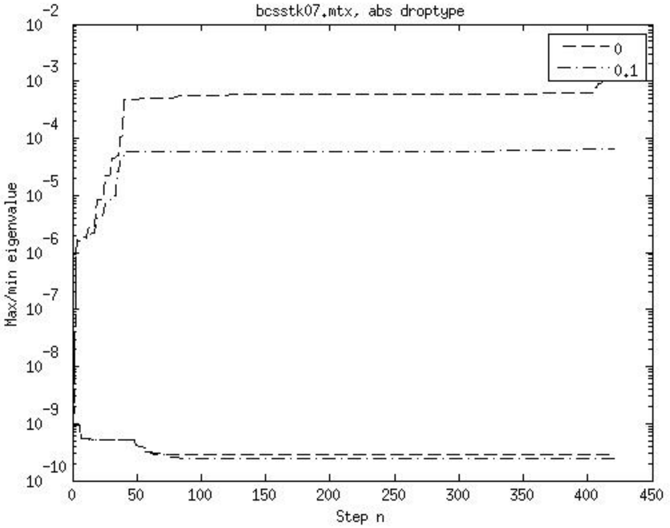
$$\alpha_{ji} = \langle z_i^{(j-1)}, z_j^{(0)} \rangle_A / (\langle z_j^{(0)}, z_j^{(0)} \rangle_A)^{1/2}$$


bcsstk07.mtx, abs droptype
0
0.1
Residual norm || r ||
10^{12}
10^{10}
10^{8}
10^{6}
10^{4}
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
0
20
40
60
80
100
120
140
Iteration n


bcsstk07.mtx, abs droptype
Loss of orthodonality
Step n
0
0.1
10^{2}
10^{0}
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
10^{-12}
10^{-14}
10^{-16}
0
50
100
150
200
250
300
350
400
450


bcsstk07.mtx, abs droptype
Submatrix eigenvalues
Step n
10^-2
10^-3
10^-4
10^-5
10^-6
10^-7
10^-8
10^-9
10^-10
0
50
100
150
200
250
300
350
400
450
× 0


bcsstk07.mtx, abs droptype
0.1
Submatrix eigenvalues
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
0
50
100
150
200
250
300
350
400
450
Step n


bcsstk07.mtx, abs droptype
Max/min eigenvalue
Step n
0
0.1
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
0
50
100
150
200
250
300
350
400
450

Conclusion and open questions

Done

- analysis for the full algorithms (without dropping)

Future work

- find a connection between estimate of $\|\bar{Z}^T A \bar{Z} - I\|$ and dropping strategy for incomplete schemes
- analyze behavior of preconditioned iterative methods

Thank you for your attention!!!