TECHNICAL UNIVERSITY OF LIBEREC
Faculty of Mechatronics, Informatics
and Interdisciplinary Studies

Model Predictive Control for Demand Response of Thermostatically Controlled Loads

The Summary of the Doctoral Thesis

Study programme:	N2612 – Electrical Engineering and Informatics
Study branch:	2612V045 – Technical Cybernetics
Author:	**Ing. Nikita Zemtsov**
Supervisor:	Doc. Dr. Ing. Mgr. Jaroslav Hlava

Liberec 2018

Abstract

Increasing the share of renewable electricity generation is a characteristic feature of modern energy systems. Renewable electricity generation has important environmental benefits, however, it is also marked by significant stochasticity and its large scale integration into power grid is not possible without new methods for control of electricity consumption, new energy storage technologies and communication infrastructure. Thermostatically controlled loads represent a significant share of total electricity consumption and they are often tightly connected with large thermal storage capacities. For these reasons they can be used for controlling electricity consumption and cost effective energy storage. This motivates the focus of this thesis on advanced control algorithms for thermostatically controlled loads.

Any control requires a suitable control signal. In this thesis, an indirect control signal is used – the role of the control signal is played by variable electricity price. This concept is considered in many pilot projects both in the USA and in the EU. It has certain advantages: the customers can choose the preferred strategy for responding to the needs of the grid, so their comfort is not compromised; also there is no need to install significantly more complex interfaces for direct control of the loads and monitoring of their states. However, the design of suitable control algorithms for responding to variable prices is still a largely open problem. The thesis focuses on two aspects of this problem.

The first part of the thesis considers the control of a single large thermostatically controlled load that responds to the price signal. This load is described by a linear time varying system and a local economic model predictive controller is designed for it. This controller must account for the time varying dynamics of the controlled load. By performing local economic optimization this controller helps to balance supply and demand in the electricity grid. This part of the thesis was created within the framework of H2020 SmartNet project and it considers one of the project pilot demonstrations: heated swimming pool. The time varying character of the model of this pool is due to the changes of the heat transfer coefficient between water and air depending on the wind speed.

The second part of the thesis focuses on smaller thermostatically controlled loads. They are negligible individually, but they can play an important role if a larger population is aggregated. The structure of the proposed control system is hierarchical. Economic model predictive controller in the upper level responds to varying electricity price and changes the temperature setpoints of the thermostats in the lower level. The objective of the control system is the same as in the first part of the thesis: the cost of the operation of this population is minimized and this helps to keep the balance in the grid. However, the high number of the loads does not allow individual modelling of each load in the model predictive controller and an aggregate model had to be developed and tested. This model is non-linear and economic model predictive controller has to solve mixed integer non-linear optimization problem. The effectiveness of the proposed control strategy was demonstrated by simulation.

Keywords: Smart Grids, Demand Response, Real Time Pricing, Economic model predictive control, Non-linear model predictive control, Modelling of aggregated thermostatically controlled loads, Linear parameter-varying systems

Contents

List of Abbreviations **7**

List of Symbols **8**

1 Introduction **11**

1.1 Smart energy grids 12

1.2 Demand side management 12

1.3 Model Predictive Control in Smart grids 14

1.4 Potential of TCLs in future energy systems 15

2 Objectives of the thesis **16**

3 E-MPC based on linear parameter-varying model **17**

3.1 Model of swimming pool heating system 18

3.2 E-MPC based on LPV model 19

3.2.1 Optimization problem 19

3.2.2 Corresponding linear program 20

3.3 Simulation results 23

4 Simulation model of TCLs population **25**

4.1 Model of an individual system with an electrical space heater 25

4.2 Model of population of electrical space heaters 26

4.3 Simulation results 27

5 Aggregate model of the population **29**

5.1 Homogeneous model 29

5.2 Heterogeneous model 33

5.3 Simulation results 34

6 E-NMPC for the population of TCLs **36**

6.1 Continuous-time optimization problem 37
6.2 Continuous-discrete-time optimization problem 37
6.3 Simulation results 38

The summary **42**

References **44**

Curriculum Vitae **50**

List of Publications **51**

List of Abbreviations

CHP Combined Heat and Power. 12

COP Coefficient of Performance. 17, 18

DER Distributed Energy Resource. 12

DLC Direct Load Control. 13

DSM Demand Side Management. 11–13, 15

E-MPC Economic Model Predictive Control. 16, 17, 23, 25, 42, 43

E-NMPC Economic Non-linear Model Predictive Control. 16, 36, 38

LPV Linear Parameter-Varying. 15–17, 19, 23, 42

LTI Linear Time Invariant. 15, 17, 18, 20

MILP Mixed-Integer Linear Program. 23

MPC Model Predictive Control. 11, 14, 15, 17, 19, 42

NMPC Non-linear Model Predictive Control. 37

PID Proportional Integral Derivative. 14

RES Renewable Energy Source. 11, 12, 14

SG Smart Grid. 12

TCL Thermostatically Controlled Load. 11, 15–17, 25, 29, 36, 42, 43

VPP Virtual Power Plant. 12

List of Symbols

Symbol	Description	Unit
A_p	pool area	m^2
A_{hom}	state matrix of the homogeneous model	
A	state matrix of the aggregate model	
C_m	mean thermal capacitance of the TCLs	$kWh/°C$
C_w	specific heat of water	$kWh/(kg\cdot°C)$
C_{agg}	output matrix of the aggregate model	
C_{bin}	output matrix of the bin state transition model	
C	thermal capacitance	$kWh/°C$
H	hysteresis band of the thermostat	$°C$
N_{bin}	total number of bins per state	
N_{high}	number of bins within high temperature range per state	
N_{low}	number of bins within low temperature range per state	
N_{norm}	number of bins within normal temperature range per state	
N	predictive horizon	
P_m	mean electrical power of the TCLs	kW
P_{nom}	nominal electric power of the heat pump	kW
P_n	normalized electrical power of the population	
P	electrical power	kW
R_m	mean thermal resistance of the TCLs	$°C/kW$
R	thermal resistance	$°C/kW$
T_0	initial temperature	$°C$

Symbol	**Description**	**Unit**
T_p	swimming pool temperature	$°C$
T_{amb}	ambient temperature	$°C$
$T_{i,low}$	lower temperature boundary of the i-th bin	$°C$
$T_{i,up}$	upper temperature boundary of the i-th bin	$°C$
T_{low}	lower boundary of the working temp. range	$°C$
T_{sp}	temperature setpoint	$°C$
T_{up}	upper boundary of the working temp. range	$°C$
T	temperature controlled by the TCL	$°C$
V_p	pool volume	m^3
V	state of the heating system	
X_0	initial state vector of the aggregate model	
X	state vector of the aggregate model	
Y	output of the aggregate model	
ΔT_{bin}	temperature range corresponding to one bin	$°C$
ΔT_{low}	difference between the lower boundary of the working temperature range and the temperature setpoint range	$°C$
ΔT_{sp}	temperature setpoint change	$°C$
ΔT_{up}	difference between the upper boundary of the working temperature range and the temperature setpoint	$°C$
α	heat transfer coefficient	
ρ_w	water density	kg/m^3
σ_{rel}	relative standard deviation	
θ	set of parameters defining a TCL unit	
m_0	initial state of the TCL	
m	state of the TCL	
n_c	number of clusters	
n_i	number of units in i-th cluster	
n	number of units in the population	
p	normalized electricity price forecast	
r_{sw}	swithing rate	
r	transition rate	

Symbol	Description	Unit
t_0	current time	sec
t_f	predictive horizon	sec
t_s	sampling time	sec
t	transition time	sec
u_{opt}	optimal profile of setpoint changes	
v_w	wind speed	m/s
w	weight of the output of the homogeneous model	
x	state vector of the bin state transition model	
y	output of the bin state transition model	

1 Introduction

Current national energy policies tend to replace fossil fuelled power plants with energy production from Renewable Energy Sources (RESs) in order to create more efficient and economic energy system as well as to deal with existing environmental problems [1–3]. In the European Union, the share of renewable energy production has been growing considerably in the past years [4].

There are several features specific to utilizing RES related to harvesting, transmitting, storing and consuming renewable energy. Firstly, RES are distributed, e.g. wind turbines, photovoltaic solar panels, and solar thermal units of different sizes that can be located almost anywhere and belong to anyone from government to an individual person. On one hand, it allows to reduce the transmitting capacity of the grid, because the energy source can be located closer to the consumers. On the other hand, it requires redesigning the energy system, because conventionally most of the energy systems are centralized. Secondly, renewable energy production has intermittent and uncontrollable nature that necessitates developing new advanced control and optimization methods, which are applied on the both production and consumption sides, for maintaining power balance in the grid.

Dealing with these issues imply utilizing advanced control and optimization algorithms (e.g. Model Predictive Control (MPC)) in Demand Side Management (DSM) for scheduling fossil-fuel electricity production, managing energy storage, predicting the overall energy consumption, and coordinating flexible portion of the loads (e.g. Thermostatically Controlled Loads (TCLs)). Thus, a variety of related problems arise: developing prediction models for the system components (renewable production units, flexible loads, and storage units) as well as models predicting overall production and consumption; defining the control hierarchy and algorithms that would coordinate all the components at different scales.

1.1 Smart energy grids

The concepts of Smart Grid (SG) and Virtual Power Plant (VPP) were introduced in order to deal with Distributed Energy Resources (DERs). SG aims to provide the enchantment to ensure high levels of security, quality, reliability, and availability of electric power [5]. VPP combines DERs, including RESs, to make it appear in market as a single power plant.

Smart grid focuses on improving the process of delivering electrical energy from suppliers to consumers involving usage of modern information and communication technology. Advanced power electronic devices such as smart meters and energy controllers provide possibility to gather information about producers and consumers of electrical energy. These devices are used to maximize the transmitting ability as well as to maintain the stable operation of the grids, intelligent control and self-healing.

Virtual Power Plant (VPP) is a new concept dealing with generation and management of energy based on centralized control structure, which connects, controls, and visualizes work of DERs, such as Combined Heat and Power (CHP) units, wind farms, solar parks, and etc. as well as flexible power consumers and batteries [6, 7].

Introducing VPP will allow more power to be generated locally and shared by participants without needs to transmit it over long distances at high voltage. Consumers will not be passive members of electrical grid anymore. They will be able to influence the power network and become prosumers: consumers that are capable of producing electrical energy [8]. Using distributed generators will allow them to decide whether it is more profitable to buy or to produce the electrical energy. Moreover, it will increase the stability of the power network in the regions where blackouts are usual or possible to occur.

1.2 Demand side management

Demand Side Management (DSM) aims to increase flexibility and efficiency of already existing power distribution infrastructure, which is conventionally over-designed to cope with maximum expected load peaks [9, 10].

There are two control methods applied by DSM [11]. The first method is the indirect load control. The power consumption is controlled from the customer side taking into account the real-time pricing or frequency deviation in power system. The second method is the Direct Load Control (DLC). In this case, the power consumption is controlled directly by a system operator. This method provides more precise adjusting of the consumption, but the customer's needs and preferences might be violated.

Although Direct Load Control (DLC) generally provides better ability to control the consumption, the price based (indirect) method has some advantages: there is no need to develop bi-directional communication interface and share knowledge about the end-user's environment; it is a decentralized structure with common control signal (electricity price) where each customer decides how to respond, so the customer's preferences are not compromised and the system itself is simpler; it clearly motivates customers to participate in DSM by providing economical benefits. If indirect method is used, consumer price of electricity varies dynamically in the real time. This varying price signal can either follow the prices of the short-term wholesale electricity markets or it can be constructed by the electricity retailer in any other suitable way [12–15].

Figure 1.1 contains an example of an energy system with varying electricity price, which is seen as a potential energy system in Denmark [16, 17]. Price Generator is used in the position of consumption controller and generates the optimal electricity price profile to meet the reference consumption taking into account the estimated response.

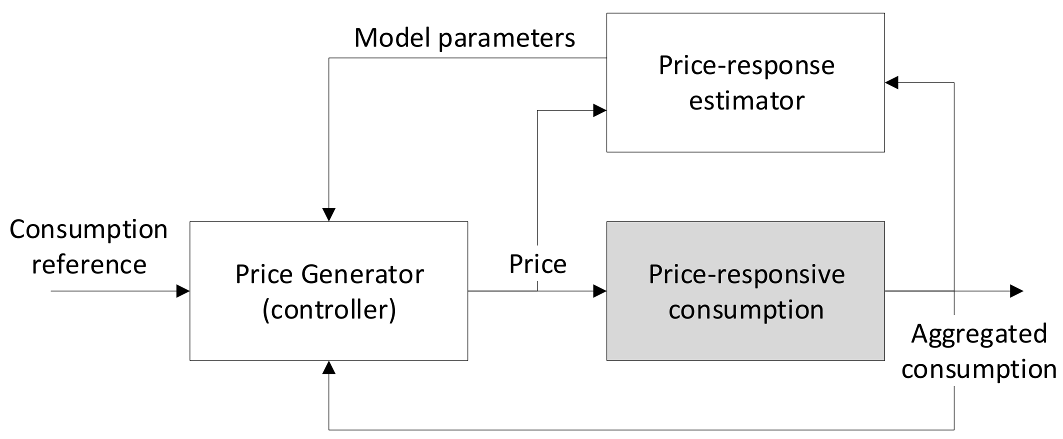

Figure 1.1: Energy system with varying electricity price signal

It is expected that the consumers will adjust their consumption to variable price in order to reduce payments for the electricity, whereas it will positively influence the grid stability (the consumption reference is generated taking into account the grid's needs).

Practical realization of such energy system requires many open questions to be answered. For instance, defining algorithms for generating consumption reference and price generator signals, developing the models for aggregate demand response approximation (price-response estimator), and etc.

In the thesis the main attention is to the flexible consumption side (price-responsive consumption). It is important to propose such optimal control strategies that will motivate the consumers to become members of the presented energy system. Ideally, the electricity payments should decrease whereas the customer comfort should not be violated. It is widely accepted in the literature that Model Predictive Control (MPC) is a well-suited method for this class of control problems [18–23].

1.3 Model Predictive Control in Smart grids

Model Predictive Control (MPC) is an advanced control technique (more advanced than classical PID controller) that has had a great success in many application in the recent decades. There are several advantages that led to that [24, 25]. Firstly, MPC allows operation near equipment and safety constraints, in the most cases it provides the most efficient or the most profitable regimes. Secondly, the method takes into account internal interaction within the controlled process using a model. In general, it is preferable to use a linear model; however, there are modifications with non-linear models [26, 27]. Thirdly, the basic formulation can easily be extended to multivariable plants with almost no modifications. Moreover, the modern processors allow to solve such optimization problems in real time.

In modern energy systems, MPC helps to maintains the power balance between production and consumption of the electrical energy [18, 19, 22], which is required due to uncontrollable nature of Renewable Energy Sources (RESs) discussed earlier.

The dynamics models of common energy system components such as electrical vehicles, heating or refrigerating systems, wind farms, solar collectors and heat storage tanks, are known [28–32], which makes MPC even more attractive for these control tasks.

This thesis is focused mainly on optimization of the consumption side, particularly Thermostatically Controlled Loads (TCLs), in order to match the current energy production forecast.

1.4 Potential of TCLs in future energy systems

Thermostatically Controlled Loads (TCLs) is a common class of energy loads that maintain temperature regulation. It is widely accepted that TCLs have enormous potential for regulation services provision due to its inherent thermal capacitance, ability of being turned OFF/ON for some period of time without compromising customer's comfort, and widely-spread usage [11, 33–37]. For example, TCLs represent about 20% of total electrical consumption in the United States [38, 39]. Moreover, it has been shown that aggregate control of TCLs is more cost-effective than other energy storage technologies such as flywheels, Li-ion, advanced lead acid, and Zinc Bromide batteries [40]. These facts make TCLs an attractive target for DSM.

There are many proposed MPC-based control systems for utilizing TCLs in DSM [18–23]. However, in these examples the scope of optimization is limited either to a single building, to relatively small groups of buildings, or to microgrids with quite modest set of power generation and consumption devices. Consequently, the appliances, generators etc. could be described by individual though simplified dynamic models (in most cases Linear Parameter-Varying (LPV) models). MPC is then formulated as a relatively simple linear or linear mixed integer program. A part of this thesis deals with extending classical MPC approach to deal with Linear Time Invariant (LTI) models.

Another approach is to focus on a large population of TCLs. Consequently it is no longer possible to model each appliance by its individual model, but suitable aggregate population model must be developed and used instead. Various models describing aggregate demand of a population of TCLs have been presented in the literature with its advantages and disadvantages [29, 32, 39, 41–44].

This work also deals with modifying bin state transition model (relatively accurate aggregate model for TCL) [43] such that it can be used for indirect control of TCLs. The original model provide relatively accurate demand response to a switching signal (direct load control), whereas the energy system considered in this thesis assumes indirect (electricity price-based) load control.

2 Objectives of the thesis

This thesis deals with developing advanced control algorithms for utilizing the potential of Thermostatically Controlled Loads (TCLs) in smart energy grid with variable electricity price.

The focus is on the price-responsive consumer's side, in particular on the optimal control algorithms that can be applied on the consumer's side, whereas the price generation algorithms are out of the thesis scope. The main objective of the proposed Economic Model Predictive Control (E-MPC) strategies is to minimize the operational cost of the flexible loads, in particular TCLs, taking into account the current electricity price and future electricity price profile.

The thesis is divided into two parts. The first part deals with optimizing energy consumption of a system with a single relatively large TCL, which can be described by a Linear Parameter-Varying (LPV) model. The corresponding objectives can be summarized as follows:

- formulate an LPV model for E-MPC controller design;
- modify the E-MPC optimization problem to account variation of the model parameters;
- verify the E-MPC control strategy.

The second part deals with aggregate control of a population of TCLs using Economic Non-linear Model Predictive Control (E-NMPC). This task doesn't only require developing an appropriate optimal control algorithm, but also an aggregate model describing demand response of the whole population. The corresponding objectives can be summarized as follows:

- design a simulation model for verification of the control algorithm;
- develop an aggregate model of the TCLs population;
- design and verify the E-NMPC control strategy.

3 E-MPC based on linear parameter-varying model

Some systems with TCLs are better described by Linear Parameter-Varying (LPV) rather than Linear Time Invariant (LTI) model because the model parameters might depend on the environmental conditions. For example, Coefficient of Performance (COP) usually depends on the ambient temperature; thermal conductivity and heat transfer coefficients may depend on the wind speed; and some other parameters may depend on the occupancy status in case of residential applications [45–47].

An example of such system is studied in the Danish Pilot, which is also a part of the SmartNet [48] and the CITIES [49] projects. The aim of the Danish Pilot is to explore the potential of aggregate control of summer houses with swimming pools to be a flexible consumer and store energy harnessed from renewable energy sources. These summer houses consume relatively high amount of energy for the swimming pool temperature and humidity control. At the same time the swimming pools, filled with water, have large thermal mass, which allows to shift and to schedule the heating profile without compromising the occupants comfort in order to optimize the energy consumption.

Classical Model Predictive Control (MPC) requires an LTI model describing plant dynamics. In order to take into account possible variation of the plant model parameters, an MPC based on an LPV model should be developed. This task was divided into the following steps:

- formulate an LPV model of the swimming pool heating system (heat pump);
- formulate an optimal control strategy taking into account variation of the model parameters (E-MPC for LPV model);
- verify the control algorithm (simulations).

3.1 Model of swimming pool heating system

Figure 3.1 contains the structure of the swimming pool heating system. The pump circulates the water in the system. The heat pump and the heat exchanger provide hot water to the system. The heat pump is assumed to be controlled externally.

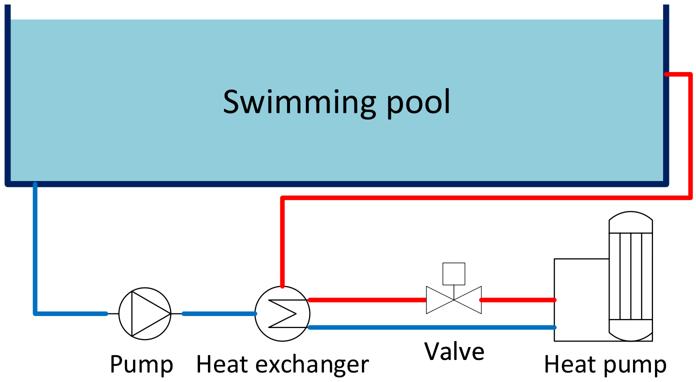

Figure 3.1: Structure of swimming pool heating system

The proposed LTI model is developed under the following assumptions:

- evaporation is neglected, thus the volume of the water in the pool is constant;
- all heat losses are lumped to a single heat loss, which depends on the ambient temperature and the wind speed;
- relationship between COP and T_{amb} is linear [50] within the considered range:

$$COP = 0.0952 \cdot T_{amb} + 3.1. \tag{3.1}$$

The model is presented below (more detailed derivation of the model is presented in the full version of thesis):

$$\rho_w \cdot C_w \cdot V_p \cdot \dot{T}_p = \alpha A_p \left(T_{amb} - T_p\right) + V \cdot P_{nom} \cdot COP, \tag{3.2a}$$

$$\alpha = \mu + \eta \cdot v_w, \tag{3.2b}$$

where T_p is the swimming pool temperature $[°C]$; ρ_w is the water density $[kg/m^3]$; C_w is the specific heat of water $[kWh/(kg \cdot^\circ C)]$; V_p is the pool volume $[m^3]$; A_p is the pool area $[m^2]$; T_{amb} is the ambient temperature $[°C]$; P_{nom} is the nominal electric power of the heat pump $[kW]$; V is the state of the heating system; μ and η are the coefficients defining the relationship between the wind speed (v_w) and the heat transfer coefficient α.

3.2 E-MPC based on LPV model

This section presents economic MPC based on a Linear Parameter-Varying (LPV) model and describes its transformation to the standard linear program. Note that these results were published in 2017 and presented in [51].

Model 3.2 can be discretized and generalized assuming that any of the parameters can vary as follows:

$$x_{k+1} = A_d(\theta_k)x_k + B_d(\theta_k)u_k + E_d(\theta_k)d_k \tag{3.3a}$$

$$y_k = C_d(\theta_k)x_k \tag{3.3b}$$

Here, y_k is the output vector; x_k is the state vector; u_k is the control vector; d_k is the measured disturbance; A_d, B_d, E_d and C_d are the state matrices; θ_k is the vector of parameters influencing the state matrices.

3.2.1 Optimization problem

The objective of the economic MPC is to minimize operational cost of the system taking into account input and output constraints [18]:

$$\min_u \quad \sum_{k=0}^{N-1} c_k u_k + \rho_v v_{k+1} \tag{3.4a}$$

$$s.t. \quad x_{k+1} = A_d(\theta_k)x_k + B_d(\theta_k)u_k + E_d(\theta_k)d_k \tag{3.4b}$$

$$y_k = C_d(\theta_k)x_k \tag{3.4c}$$

$$y_{min,k} - v_k \leq y_k \leq y_{max,k} + v_k \tag{3.4d}$$

$$u_{min,k} \leq u_k \leq u_{max,k} \tag{3.4e}$$

Here, N is the prediction horizon; c_k is the cost coefficients (e.g. electricity price); v_k is the slack variables relaxing the output constraints with corresponding penalty cost ρ_v; $y_{min,k}$ and $y_{max,k}$ are the output constraints; $u_{min,k}$ and $u_{max,k}$ are the input constraints.

3.2.2 Corresponding linear program

Problem (3.4) can be converted to a standard linear optimization problem of the following form:

$$\min_{x} \quad f^T x \tag{3.5a}$$

$$s.t. \quad A_{ineq} x \leq b_{ineq} \tag{3.5b}$$

$$x_{min} \leq x \leq x_{max} \tag{3.5c}$$

Here, f is the vector of cost coefficients; x is the vector of variables to be determined; A_{ineq} and b_{ineq} define the inequality constraints; x_{min} and x_{max} are the minimum and maximum boundaries of x respectively.

The transformation from optimal problem (3.4) to optimal problem (3.5) is presented below. Table 3.1 contains corresponding notations. The transformation is based on similar transformation for the case of the LTI model presented in [25, 52]

Table 3.1: MPC for LPV model notations

Parameter	Description
$U = [u_0\ u_1\ \dots\ u_{N-1}]^T$	Vector of future inputs
$Y = [y_1\ y_2\ \dots\ y_N]^T$	Vector of predicted outputs
$U_{min} = [u_{min,0}\ \dots\ u_{min,N-1}]^T$	Vector of min. input constraints
$U_{max} = [u_{max,0}\ \dots\ u_{max,N-1}]^T$	Vector of max. input constraints
$Y_{min} = [y_{min,1}\ \dots\ y_{min,N}]^T$	Vector of min. output constraints
$Y_{max} = [y_{max,1}\ \dots\ y_{max,N}]^T$	Vector of max. output constraints
$D = [d_0\ d_1\ \dots\ d_{N-1}]^T$	Vector of measured disturbances
$\Theta = [\theta_0\ \theta_2\ \dots\ \theta_N]^T$	Vector of parameters
$C = [c_0\ c_1\ \dots\ c_{N-1}]^T$	Vector of cost coefficients
$V = [v_0\ v_1\ \dots\ v_{N-1}]^T$	Vector of slack variables
$P = [\rho_v\ \rho_v\ \dots\ \rho_v]^T$	Vector of penalty cost coefficients

Equations (3.6) and (3.7) demonstrate calculation of state and output vectors predictions with given initial states x_0, future control inputs U, and measured disturbances D. Where $A_k = A_d(\theta_k)$, $B_k = B_d(\theta_k)$, $E_k = E_d(\theta_k)$ and $C_k = C_d(\theta_k)$.

$$x_1 = A_0 x_0 + B_0 u_0 + E_0 d_0 \tag{3.6a}$$

$$\begin{aligned} x_2 &= A_1 x_1 + B_1 u_1 + E_1 d_1 \\ &= A_1(A_0 x_0 + B_0 u_0 + E_0 d_0) + B_1 u_1 + E_1 d_1 \\ &= A_1 A_0 x_0 + A_1 B_0 u_0 + B_1 u_1 + A_1 E_0 d_0 + E_1 d_1 \end{aligned} \tag{3.6b}$$

$$\begin{aligned} x_3 &= A_2 x_2 + B_2 u_2 + E_2 d_2 \\ &= A_2(A_1 A_0 x_0 + A_1 B_0 u_0 + B_1 u_1 + A_1 E_0 d_0 + E_1 d_1)+ \\ &\quad + B_2 u_2 + E_2 d_2 \\ &= A_2 A_1 A_0 x_0 + A_2 A_1 B_0 u_0 + A_2 B_1 u_1 + B_2 u_2 + \\ &\quad + A_2 A_1 E_0 d_0 + A_2 E_1 d_1 + E_2 d_2 \end{aligned} \tag{3.6c}$$

$$\begin{aligned} &\quad ... \\ x_k = &\left(\prod_{i=0}^{k-1} A_i\right) x_0 + \sum_{i=0}^{k-1} \left(\prod_{j=k-1}^{i+1} A_j\right) B_i u_i + \\ &+ \sum_{i=0}^{k-1} \left(\prod_{j=k-1}^{i+1} A_j\right) E_i d_i \end{aligned} \tag{3.6d}$$

$$\begin{aligned} y_k = \; & C_k \left(\prod_{i=0}^{k-1} A_i\right) x_0 + C_k \sum_{i=0}^{k-1} \left(\prod_{j=k-1}^{i+1} A_j\right) B_i u_i + \\ & + C_k \sum_{i=0}^{k-1} \left(\prod_{j=k-1}^{i+1} A_j\right) E_i d_i \end{aligned} \tag{3.7}$$

$$\begin{aligned} \underbrace{\begin{bmatrix} y_1 \\ y_2 \\ ... \\ y_N \end{bmatrix}}_{Y} = \underbrace{\begin{bmatrix} O_1 \\ O_2 \\ ... \\ O_N \end{bmatrix}}_{\Phi} x_0 + \underbrace{\begin{bmatrix} H_{u,1,0} & 0 & ... & 0 \\ H_{u,2,0} & H_{u,2,1} & ... & 0 \\ ... & ... & ... & ... \\ H_{u,N,0} & H_{u,N,1} & ... & H_{u,N,N-1} \end{bmatrix}}_{\Gamma_u} \underbrace{\begin{bmatrix} u_0 \\ u_1 \\ ... \\ u_{N-1} \end{bmatrix}}_{U} \\ + \underbrace{\begin{bmatrix} H_{d,1,0} & 0 & ... & 0 \\ H_{d,2,0} & H_{d,2,1} & ... & 0 \\ ... & ... & ... & ... \\ H_{d,N,0} & H_{d,N,1} & ... & H_{d,N,N-1} \end{bmatrix}}_{\Gamma_d} \underbrace{\begin{bmatrix} d_0 \\ d_1 \\ ... \\ d_{N-1} \end{bmatrix}}_{D} \end{aligned} \tag{3.8}$$

Output vector predictions (3.7) can be rewritten in a shorter form:

$$y_k = O_k x_0 + \sum_{i=0}^{k-1} H_{u,k,i} u_i + \sum_{i=0}^{k-1} H_{d,k,i} d_i \tag{3.9}$$

Here O_k is the extended observability matrix:

$$O_k = C_k \left(\prod_{i=0}^{k-1} A_i \right) \tag{3.10}$$

Here H_u and H_d are the Markov parameters with respect to manipulated variable and measured disturbance:

$$H_{u,k,i} = C_k \left(\prod_{j=k-1}^{i+1} A_j \right) B_i \tag{3.11a}$$

$$H_{d,k,i} = C_k \left(\prod_{j=k-1}^{i+1} A_j \right) E_i \tag{3.11b}$$

The predicted output vector Y is calculated using (3.9):

$$Y = \Phi x_0 + \Gamma_u U + \Gamma_d D \tag{3.12}$$

see equation (3.8) for detailed structures of matrices Φ, Γ_u, and Γ_d.

The input constraints are:

$$U_{min} \leq U \leq U_{max} \tag{3.13}$$

The output constraints are:

$$Y_{min} - V \leq Y \leq Y_{max} + V \tag{3.14}$$

Using (3.12), (3.14) can be rewritten as:

$$Y_{min} - V \leq \Phi x_0 + \Gamma_u U + \Gamma_d D \leq Y_{max} + V \tag{3.15a}$$

$$\Gamma_u U - V \leq Y_{max} - \Phi x_0 - \Gamma_d D \tag{3.15b}$$

$$-\Gamma_u U - V \leq -Y_{min} + \Phi x_0 + \Gamma_d D \tag{3.15c}$$

Finally, the parameters of the standard linear optimization problem (3.5) can be found as following:

$$x = \begin{bmatrix} U \\ V \end{bmatrix} \tag{3.16a}$$

$$f = \begin{bmatrix} C \\ P \end{bmatrix} \tag{3.16b}$$

$$A_{ineq} = \begin{bmatrix} \Gamma_u & -I \\ -\Gamma_u & -I \\ 0 & -I \end{bmatrix} \tag{3.16c}$$

$$b_{ineq} = \begin{bmatrix} Y_{max} - \Phi x_0 - \Gamma_d D \\ -Y_{min} + \Phi x_0 + \Gamma_d D \\ 0 \end{bmatrix} \tag{3.16d}$$

$$x_{min} = \begin{bmatrix} U_{min} \\ -\infty \end{bmatrix} \tag{3.16e}$$

$$x_{max} = \begin{bmatrix} U_{max} \\ +\infty \end{bmatrix} \tag{3.16f}$$

Note that in some systems, including the swimming pool heating system considered in Section 3, the manipulated variables can only have two states (”OFF”/”ON”). The technique proposed in this section can still be applied for E-MPC design by adding the following constrain: $U \in \{0, 1\}$. Then (3.5) becomes a Mixed-Integer Linear Program (MILP) optimization problem.

3.3 Simulation results

This section contains simulation results that verify the proposed E-MPC based on the LPV model. The control strategy is able to take into account the influence of the environmental conditions (wind speed and ambient temperature) on the swimming pool water mass thermodynamics. Also the occupancy status is taken into account by adjusting the temperature constraints. The exact simulation parameters are presented in the full version of thesis.

Figure 3.2 demonstrates the expected behavior: the system consumes the least amount of energy possible (the controlled temperature is kept as close as possible to the lower limit); the heating profile is scheduled such that the system is in ON state during the times of lowest electricity prices.

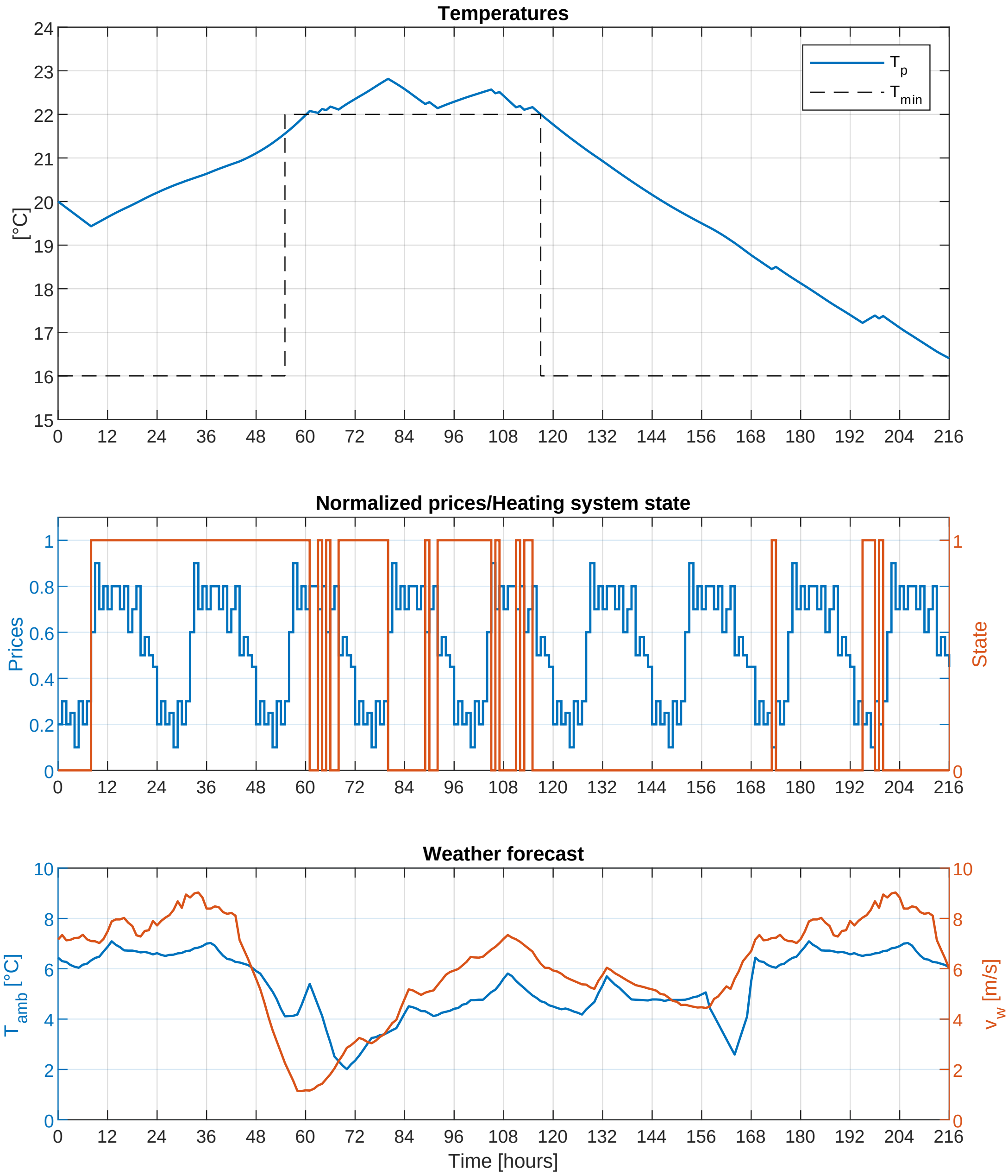

Figure 3.2: Simulation results: swimming pool heating system

4 Simulation model of TCLs population

The second part of the thesis develops an economically optimal control system for a population of Thermostatically Controlled Loads (TCLs) in energy system with variable and predictable electricity price. The simulation model is developed to verify the control system and control algorithm. The main requirement for the simulation model is to provide accurate and realistic dynamics of the population, which is achieved by simulating each unit individually. A population of electrical space heaters is considered as a case study.

4.1 Model of an individual system with an electrical space heater

An electrical space heater is a Thermostatically Controlled Load (TCL) with hysteresis control law: the load is turned ON when the controlled temperature is below the lower hysteresis boundary and turned OFF when the controlled temperature is above the higher hysteresis boundary. When turned ON, the unit consumes electrical power, whereas in OFF state the consumption is equal to zero.

It is assumed that the temperature setpoint can be externally changed in order to conform with the control system. This feature provides an interface for the higher level E-MPC to indirectly influence the state of the unit.

The model of a single unit of the population is based on the model presented in [53] and is formulated as follows:

$$\frac{dT(t)}{dt} = -\frac{1}{CR}[T(t) - T_{amb}(t) - m(t)RP], \tag{4.1a}$$

$$m(t^+) = \begin{cases} 0 & \textbf{if} \quad T \geq T_{sp} + \Delta T_{sp} + H \\ 1 & \textbf{if} \quad T \leq T_{sp} + \Delta T_{sp} - H \\ m(t) & \qquad \textbf{otherwise} \end{cases} \tag{4.1b}$$

Here, T is the temperature controlled by the TCL [$°C$]; C is the thermal capacitance [$kWh/°C$] and R the is thermal resistance [$°C/kW$] of the heated area (e.g. room); P is the electrical power [kW]; T_{amb} is the ambient temperature [$°C$]; $m \in \{0, 1\}$ is the state of the TCL (OFF anf ON respectively); T_{sp} is the temperature setpoint [$°C$]; ΔT_{sp} is the temperature setpoint change [$°C$]; H is the hysteresis band of the thermostat [$°C$].

Separated T_{sp} and ΔT_{sp} signals allow to have a single manipulated variable (ΔT_{sp}) to control all the units in the population, whereas each unit can have an individual T_{sp} specified by the customer.

4.2 Model of population of electrical space heaters

The simulation model provides demand response of the whole population to the temperature setpoint change (ΔT_{sp}) and the ambient temperature (T_{amb}).

The population consist of n heating systems with electrical space heaters, each unit of the population is described by an individual set of parameters $\theta_i = [C_i,\ R_i,\ P_i,\ T_{sp,i},\ H,\ T_{0,i},\ m_{0,i}]$ and simulated according to (4.1). Therefore the model is highly accurate, but the complexity of the model depends on the number of units in the population, that is why this model is not suitable for model based control system design.

The thermal capacitances (C_i), the thermal resistances (R_i), and the electrical powers (P_i) of units are log-normally distributed with the corresponding means (C_m, R_m, and P_m) and relative standard deviation (σ_{rel}). Log-normal distribution guaranties that these parameters never take negative value, which corresponds to the realistic scenario; whereas it is shown in [53] that the type of distribution doesn't have a significant impact on the demand response.

The hysteresis band of the thermostats (H) is the same for all units; the temperature setpoints ($T_{sp,i}$) are evenly distributed within $[T_{low} + H,\ T_{up} - H]$, here $[T_{low},\ T_{up}]$ corresponds to the working temperature range of the population; the initial temperatures ($T_{0,i}$) are randomly chosen from $[T_{sp,i} - H,\ T_{sp,i} + H]$.

The output of the model, the aggregate normalized demand of the whole population, is given by the following ratio:

$$P_n(t) = \frac{\sum_1^n m_i(t) P_i}{\sum_1^n P_i} \tag{4.2}$$

4.3 Simulation results

This section presents simulation results of the whole population. Table 4.1 contains values of the population parameters used for simulation and derived from typical building stock in the Czech Republic [54]. The ambient temperature (T_{amb}) is constant.

Table 4.1: Population parameters

Par.	Value	Units	Description
R_m	3.4	$°C/kW$	mean thermal resistance of the TCLs
C_m	7	$kWh/°C$	mean thermal capacitance of the TCLs
P_m	13.4	kW	mean electrical power of the TCLs
σ_{rel}	0.2, 0.5		relative standard deviation
H	0.5	$°C$	hysteresis band of the thermostat
T_{low}	20	$°C$	lower boundary of the working temp. range
T_{up}	24	$°C$	upper boundary of the working temp. range
T_{amb}	-5	$°C$	ambient temperature
n	10000		number of units in the population

Figures 4.1 and 4.2 contain the simulation results. The population can either be balanced (constant consumption) or imbalanced (changing consumption). When the population is balanced the temperature controlled by the TCLs ($T_{,i}$) are distributed such that ratio between ON and OFF units is a constant. Increasing/decreasing the temperature setpoint change (ΔT_{sp}) leads to the changing the ON/OFF cycles of the units, therefore the disbalance represented by the oscillations in demand response occurs. The figures also show that heterogeneity level (σ_{rel}) has a significant influence on the dynamics of the population. Moreover, the demand response has a non-linear nature which can be explained by non-linear nature of the hysteresis control principle.

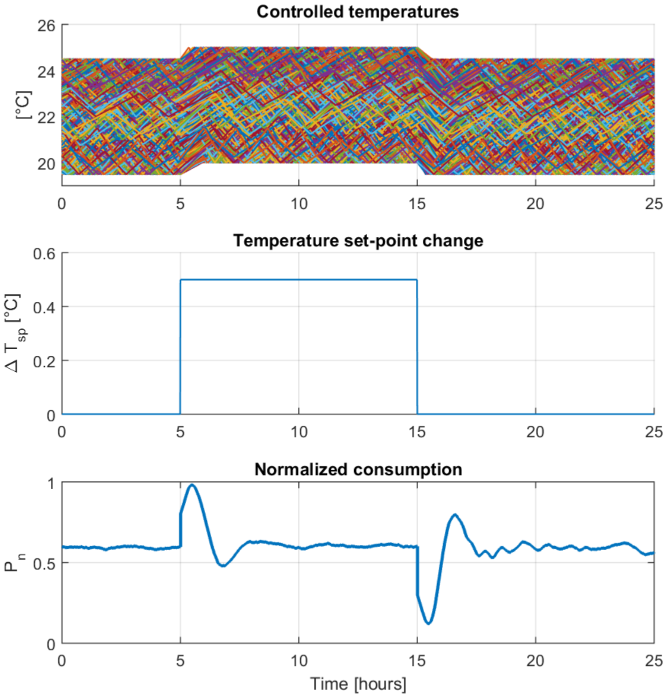

Figure 4.1: Population: temperature setpoint change test, $\sigma_{rel} = 0.2$

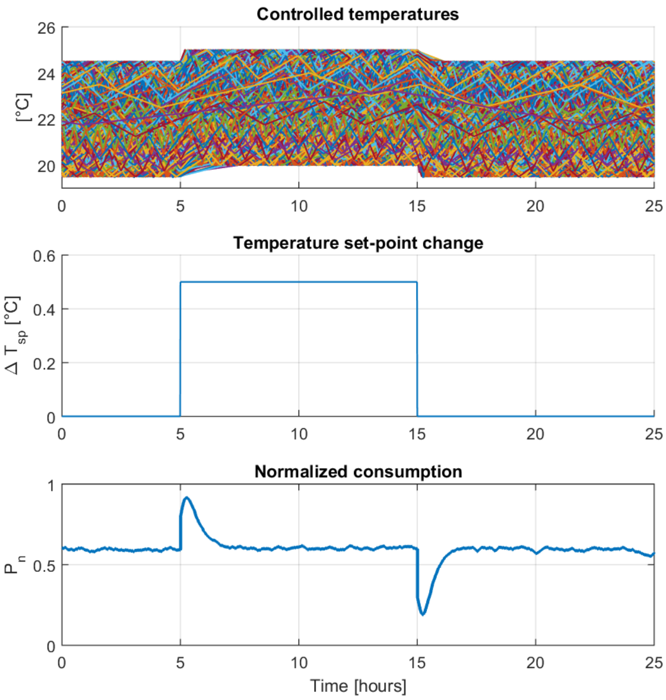

Figure 4.2: Population: temperature setpoint change test, $\sigma_{rel} = 0.5$

5 Aggregate model of the population

The simulation model presented in Chapter 4 is computationally too intensive to be utilized for model-based control of a large population of TCLs. The existing aggregate models cannot provide relatively accurate demand response of the population of TCLs to a temperature setpoint change.

The aggregate model presented in this chapter is based on the bin state transition model presented in [43, 44, 55]. The original model describes evolution of the units states and implies direct control of the population: the input signal of the model allows to manipulate the loads states; such approach have some disadvantages discussed earlier.

The proposed aggregate model is a non-linear modification of the bin state transition model which allows to control the population by changing the temperature setpoint (indirectly influencing the load states). The model consists of two parts: homogeneous and heterogeneous models.

5.1 Homogeneous model

The proposed non-linear modification of bin state transition model, unlike the original model, approximates the demand response of the population to temperature set-point change.

The original bin state transition model is used for approximating demand response of a large population of TCLs assuming that their states can be directly controlled. In the original formulation it is assumed that the population is homogeneous: the loads are defined by identical set of parameters $\theta = [C,\ R,\ P,\ T_{sp},\ H]$. The operating temperature range $[T_{low}\ T_{up}]$ is evenly divided into $2N_{bin}$ state bins, see Figure 5.1. The idea of the model is to describe distribution and natural transition of the unis over these bins. Each bin is characterized by the corresponding state, temperature range, and transitions rate to the next bin. Solid lines demonstrate transition of the unis under normal operational conditions: when the loads are turned OFF the controlled temperature decreases and vice versa; when the controlled temperature reaches one of the temperature limits $[T_{low}\ \ T_{up}]$ the corresponding loads change their state according to the hysteresis control law. Direct manipulation of the loads states is shown by dashed lines. More detailed description of bin state transition model can be found in [43, 44, 55].

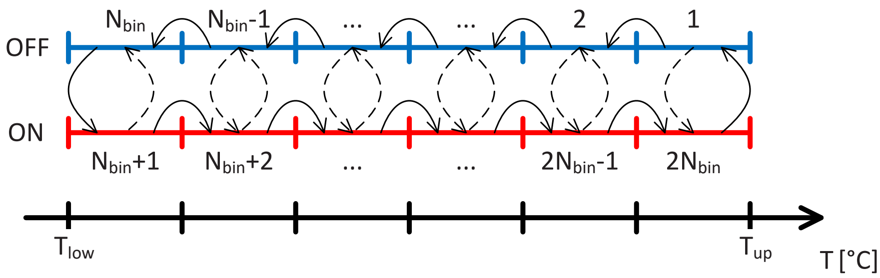

Figure 5.1: Uniform bin state transition model

The modified aggregate model implies extended operating temperature range, which does not only include the dead-band of the units (defined by the hysteresis width), but also some low- and high-temperature margins as shown in Figure 5.2. This modification allows to influence the units states indirectly, by changing the temperature setpoint. Note that the temperature axis is shifted by T_{sp} in order to demonstrate the influence of the setpoint change on the model structure. Consequently, the operating range of the modified model defines the acceptable temperature setpoint change. Note that the results with non-linear aggregate model were published in 2018 an presented in [56].

The operating range $[\Delta T_{low},\ \Delta T_{up}]$ is divided into three parts: low, normal, and high temperatures. The normal temperature range $[\Delta T_{sp} - H,\ \Delta T_{sp} + H]$ (marked by green colour) contains N_{norm} OFF and N_{norm} ON bins. The units corresponding to these bins behave according to the original bin state transition model: the solid thin lines correspond to heating or cooling process depending on the units states according to (4.1a), the dashed thin lines correspond to changing the state when a unit reachs one of the hysteresis boundaries according to (4.1b).

The low temperature range $[\Delta T_{low},\ \Delta T_{sp} - H]$ contains N_{low} OFF and N_{low} ON bins. The units corresponding to the low OFF bins behave differently: they should

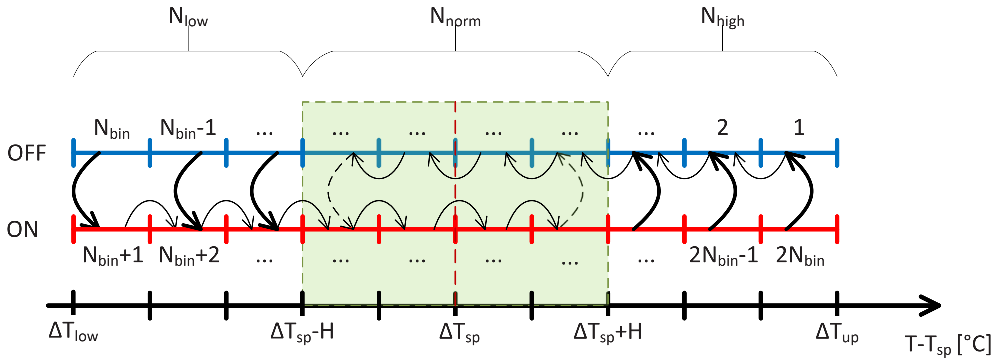

Figure 5.2: Homogeneous aggregate model

immediately change their states which means to be transferred to the corresponding ON bins (shown by thick solid lines). Such situation occurs after the setpoint has been increased and some heaters are instantly switched ON by the thermostats with accordance to (4.1b).

The high temperature range $[\Delta T_{sp} + H,\ T_{low}]$ contains N_{high} OFF and N_{high} ON bins and implies the opposite situation: the temperature setpoint has been decreased and some heaters are instantly switched OFF.

Homogeneous model (5.1) is given in a state-space form. Note that the state matrix A_{hom} is not static: its structure depends on the temperature setpoint change. In addition the model takes into account the ambient temperature according to equations (5.6) and (5.7).

$$\dot{x}(t) = A_{hom}(\Delta T_{sp}(t), T_{amb}(t))x(t) \tag{5.1a}$$

$$y(t) = C_{bin}x(t) \tag{5.1b}$$

Structure of the state matrix depends on the number of bins in the low and high temperature ranges. Elements corresponding to the **natural transition** (thin solid lines, $i \in [1 \ldots N_{high} + N_{norm} - 1,\ N_{bin} + 1 \ldots N_{bin} + N_{low} + N_{norm}]$):

$$A_{hom,i,i} = -r_i \tag{5.2a}$$

$$A_{hom,i+1,i} = r_i \tag{5.2b}$$

Switching (thin dashed lines, $i \in \{N_{high} + N_{norm},\ N_{bin} + N_{low} + N_{norm}\}$:

$$A_{hom,i,i} = -r_i \tag{5.3a}$$

$$A_{hom,2N_{bin}+1-i,i} = r_i \tag{5.3b}$$

Forced switching (thick solid lines, $i \in [N_{high} + N_{norm} + 1 \ ... \ N_{bin}, \ N_{bin} + N_{low} + N_{norm} + 1 \ ... \ 2N_{bin}]$:

$$A_{hom,i,i} = -r_{sw} \tag{5.4a}$$

$$A_{hom,2N_{bin}+1-i,i} = r_{sw} \tag{5.4b}$$

The rates are calculated taking into account the times it takes to heat or cool the controlled temperature withing the corresponding bin limits:

$$r_i = \frac{1}{t_i} \tag{5.5}$$

The times for the **OFF-bins** ($i \in [1...N_{bin}]$) are calculated as:

$$t_i = -RC\ln\left(\frac{T_{i,up} - T_{amb}}{T_{i,low} - T_{amb}}\right) \tag{5.6a}$$

$$T_{i,low} = T_{up} - (i-1)\Delta T_{bin} \tag{5.6b}$$

$$T_{i,up} = T_{up} - i \cdot \Delta T_{bin} \tag{5.6c}$$

The times for the **ON-bins** ($i \in [N_{bin} + 1 \ ... \ 2N_{bin}]$) are calculated as:

$$t_i = -RC\ln\left(\frac{T_{i,low} - T_{amb} - PR}{T_{i,up} - T_{amb} - PR}\right) \tag{5.7a}$$

$$T_{i,low} = T_{low} - (i - N_{bin} - 1)\Delta T_{bin} \tag{5.7b}$$

$$T_{i,up} = T_{low} - (i - N_{bin})\Delta T_{bin} \tag{5.7c}$$

Here, $[T_{i,low} \ \ T_{i,up}]$ defines the temperature range of the i-th bin, T_{amb} is the ambient temperature, ΔT_{bin} is the temperature range corresponding to one bin:

$$\Delta T_{bin} = \frac{\Delta T_{up} - \Delta T_{low}}{N_{bin}} \tag{5.8}$$

The modification introduces a new transition rate, which is not used in the original model and called forced switching rate (shown by the thick solid lines). It is the same for all bins and should meet the following requirement:

$$r_{sw} \gg \max\left(r_i\right), \ i \in [1...2N_{bin}] \tag{5.9}$$

The number of bins corresponding to the normal temperature ($2N_{norm}$) range does not depend on the temperature setpoint change:

$$N_{norm} = \frac{2H}{\Delta T_{bin}} \tag{5.10}$$

Whereas the number of bins corresponding to the low/high temperature ranges ($2N_{low}$ and $2N_{high}$ respectively) depends on the temperature setpoint change:

$$N_{low} = \frac{\Delta T_{low} - \Delta T_{sp} + H}{N_{bin}} \tag{5.11a}$$

$$N_{high} = \frac{\Delta T_{sp} + H - \Delta T_{up}}{N_{bin}} \tag{5.11b}$$

5.2 Heterogeneous model

In real population there are no identical units, each of them is defined by the unique set of parameters $\theta^{(i)}$, $i \in [1...n]$. The heterogeneous model deals with this variation of the parameters applying the k-means clustering method [29, 39, 57]. The population of n units is divided into into n_c clusters, each cluster is associated with the corresponding set of parameters θ_i and the number of units belonging to this cluster n_i, such that $n = \mathrm{sum}(n_i)$, $i \in [1...n_c]$.

The heterogeneous model is a combination of n_c homogeneous models defined in the previous section:

$$\dot{X}(t) = A(\Delta T_{sp}(t), T_{amb}(t))X(t) \tag{5.12a}$$

$$Y(t) = C_{agg}X(t) \tag{5.12b}$$

Here, $A \in \mathbb{R}^{2n_cN_{bin}\times 2n_cN_{bin}}$ is the state matrix of the aggregate model, $C_{aggi} \in \mathbb{R}^{1\times 2n_cN_{bin}}$ is the output matrix of the aggregate model, $X_i \in \mathbb{R}^{2n_cN_{bin}\times 1}$ is the state vector of the aggregate model, Y_i is the output of the aggregate model, which is approximation of the normalized aggregate response of the heterogeneous population (4.2).

The state vector X is given as:

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_{n_c} \end{bmatrix} \tag{5.13}$$

The state matrix A is given as:

$$A = \begin{bmatrix} A_{hom,1} & 0_{2N_{bin}} & \cdots & 0_{2N_{bin}} \\ 0_{2N_{bin}} & A_{hom,2} & \cdots & 0_{2N_{bin}} \\ \cdots & \cdots & \cdots & \cdots \\ 0_{2N_{bin}} & 0_{2N_{bin}} & \cdots & A_{hom,n_c} \end{bmatrix} \tag{5.14}$$

The output matrix C_{agg} weighs the outputs of the homogeneous models in order to normalize the output of the aggregate model to the maximum power consumption:

$$C_{agg} = \begin{bmatrix} w_1 C_{bin,1} & w_2 C_{bin,2} & \cdots & w_{n_c} C_{bin,n_c} \end{bmatrix} \tag{5.15}$$

$$w_i = \sum_{j \in \text{cluster } i} P_j / \sum_{j \in 1}^{n} P_j \tag{5.16}$$

Here, $A_{hom,i}$, x_i, and $C_{bin,i}$ are the corresponding to the i-th cluster parameters of homogeneous model, w_i is the weight of the output of the homogeneous model of the i-th cluster, $0_{2N_{bin}}$ is the zero matrix of the corresponding dimension, $\text{sum}(x_i) = n_i$, $i \in [1...n_c]$.

5.3 Simulation results

In this section the performance of the aggregate model (agg) presented in Section 5.2 is compared to the simulation model (sim) presented in Section 4.2 using the same population parameters (Table 4.1).

Figures 5.3 and 5.4 demonstrate that the aggregate model is able to approximate the dynamics of the population and to deal with heterogeneity. Since the complexity of the model depends significantly on the number of clusters (n_c) and the number of bins (N_{bin}), these values were chosen as a trade-off between complexity and accuracy of the aggregate model. Note, that higher variance of the population parameters or higher σ_{rel} requires higher number of clusters and less sensitive to the number of bins and vice versa.

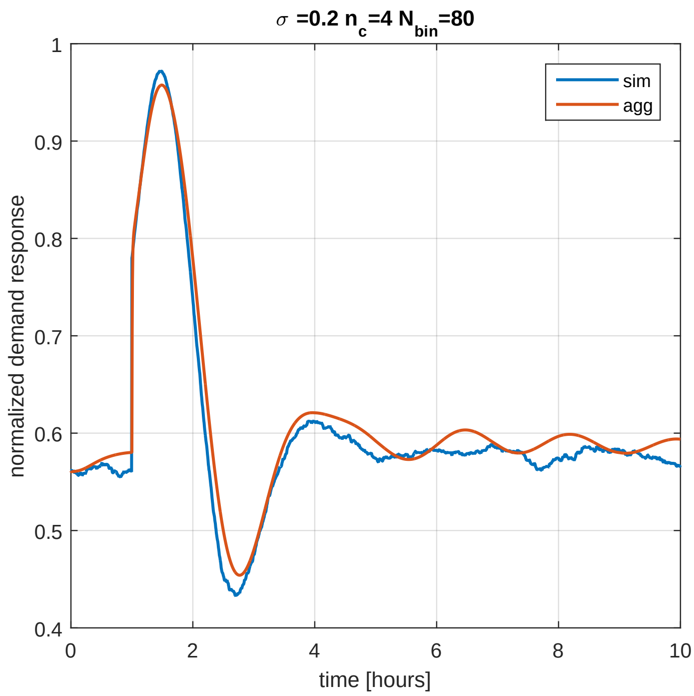

Figure 5.3: Simulation results: $\sigma_{rel} = 0.2$, $n_c = 4$, $N_{bin} = 80$

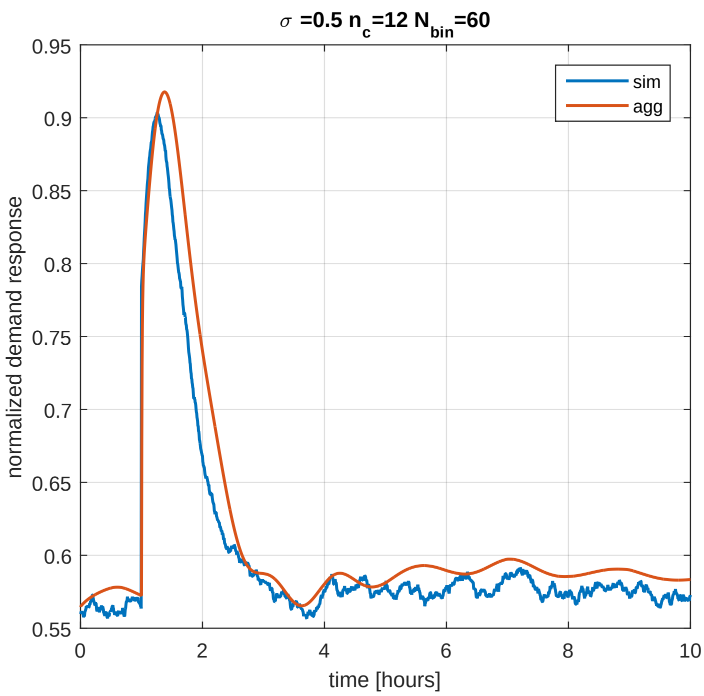

Figure 5.4: Simulation results: $\sigma_{rel} = 0.5$, $n_c = 12$, $N_{bin} = 60$

6 E-NMPC for the population of TCLs

This chapter develops a price-responsive control strategy for the population of TCLs using the aggregate model developed in Chapter 5. Figure 6.1 contains structure of the control system. The controller design is based on the idea of E-NMPC [27, 58, 59]. It is assumed that electricity price regularly changes and the electricity price forecast is available at least one day ahead. The main objective of the controller is to minimize operational cost of the whole population by shifting the electricity consumption of the population to the low-price periods.

The TCLs ON/OFF states (which are directly linked to the electrical energy consumption) are manipulated indirectly by changing temperature setpoints of the units. The optimal temperature setpoint change signal generated by the controller is constrained taking into account the customers comfort boundaries. Information about the population (current values of the controlled temperature, the units states, and total electrical power consumption) is used as a feedback signal.

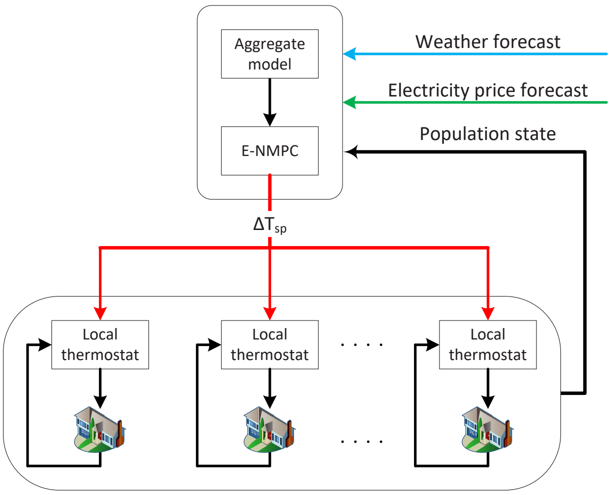

Figure 6.1: E-NMPC control system structure

6.1 Continuous-time optimization problem

The control problem (6.1) is formulated as a constrained continuous-time optimization problem, because the developed non-linear aggregate model in Chapter 5 is also presented in continuous-time domain. Moreover, Non-linear Model Predictive Control (NMPC) is often formulated as a continuous-time optimization problem [27, 58, 59].

$$
\begin{aligned}
&\min_{\Delta T_{sp}} \psi = \int_{t_0}^{t_f} p(t)Y(t)dt && \text{(6.1a)}\\
&s.t. \quad \dot{X}(t) = A(\Delta T_{sp}(t), T_{amb}(t))X(t) && \text{(6.1b)}\\
&\qquad Y(t) = C_{agg}X(t) && \text{(6.1c)}\\
&\qquad X(t_0) = X_0 && \text{(6.1d)}\\
&\qquad \Delta T_{low} \leq \Delta T_{sp}(t) \leq \Delta T_{up} && \text{(6.1e)}\\
&\qquad \frac{T_{sp}(t)}{\Delta T_{bin}} \in \mathbb{Z} && \text{(6.1f)}\\
&\qquad t \in [t_0 \; t_f] && \text{(6.1g)}
\end{aligned}
$$

Here, t_0 is the current time; t_f is the predictive horizon; p is the normalized electricity price forecast; Y is the predicted normalized electrical consumption by the whole population; ΔT_{low}, ΔT_{up} correspond to the comfort constraints; X_0 is the initial state vector of the aggregate model.

Cost function (6.1a) represents predicted normalized operational cost of the population for given ΔT_{sp} profile. Aggregate model described by (6.1b) and (6.1c) is the model defined in Section 5.2 used for calculating the normalized demand predictions.

6.2 Continuous-discrete-time optimization problem

The solution of optimization problem (6.1) is infinite-dimensional. Therefore, it is needed to be converted into a finite-dimensional continuous-discrete-time problem by approximating the input and disturbance profiles by corresponding piece-wise constant profiles:

$$\Delta T_{sp,k} = \Delta T_{sp}(t)\ t_k \leq t \leq t_{k+1} \tag{6.2a}$$

$$T_{amb,k} = T_{amb}(t)\ t_k \leq t \leq t_{k+1} \tag{6.2b}$$

$$p_k = p(t)\ t_k \leq t \leq t_{k+1} \tag{6.2c}$$

Consequently the resulting continuous-discrete-time optimization problem is given as:

$$\min_{\Delta T_{sp}} \psi = \sum_{k=1}^{N} \int_{t_k}^{t_{k+1}} p_k Y(t) dt \tag{6.3a}$$

$$s.t.\quad \dot{X}_i(t) = A(\Delta T_{sp,k}, T_{amb,k}) X(t) \tag{6.3b}$$

$$Y(t) = C_{agg} X(t) \tag{6.3c}$$

$$X(t_0) = X_0 \tag{6.3d}$$

$$\Delta T_{low} \leq \Delta T_{sp,k} \leq \Delta T_{up} \tag{6.3e}$$

$$\frac{\Delta T_{sp,k}}{\Delta T_{bin}} \in \mathbb{Z} \tag{6.3f}$$

$$t \in [t_0\ t_f] \tag{6.3g}$$

$$k \in [0\ N] \tag{6.3h}$$

The predictive horizon $[t_0\ t_f]$ is divided into N steps with sampling time t_s.

The solution of problem (6.3) is the optimal profile of temperature setpoint changes:

$$u_{opt}(t_0) = [u_0,\ u_1,\ ...,\ u_{N-1}]^T,\ u_{opt}(t_0) \in \mathbb{R}^{N\times 1} \tag{6.4}$$

At each step the first entry of the profile is implemented on the process and kept during the following sampling time.

The obtained optimization problems is solved using the gradient descent method, more details are presented in the full version of the thesis.

6.3 Simulation results

Two different scenarios were simulated in order to verify the aggregate model and E-NMPC algorithm. First, 24 hours scenario was simulated: the algorithm aims to minimize the operational cost of the whole population for the given electricity price and ambient temperature profiles. Table 6.1 contains parameters of the aggregate

Table 6.1: Aggregate model and controller parameters

Par.	Value	Units	Description
σ_{rel}	0.2		Relative standard deviation
ΔT_{low}	-2	°C	Lower limit on temperature setpoint change
ΔT_{up}	2	°C	Upper limit on temperature setpoint change
n_c	4		Number of clusters
N_{bin}	80		Number of bins
t_s	1	hour	Sampling time of the controller
N	12		Predictive horizon
ΔT_{bin}	0.5	°C	Minimum increment of setpoint change

model and the controller. Figure 6.2 contains the obtained simulation results: higher electricity price corresponds to the lower energy consumption and vice versa; the setpoint change tends to the lower limit which corresponds to the economically optimal operating regime. Moreover, the algorithm overheats the population during low-price periods in order to reduce the consumption during high-price periods.

Secondly, two energy saving strategies and zero temperature setpoint change strategy ($\Delta T_{sp} = 0$, which basically means that there is no external influence on the population) were compared in order to demonstrate the performance of the designed E-NMPC control system. The first strategy is the smart energy saving (smart) which implies that the optimal temperature setpoint change is provided by the E-NMPC. The second is the thrifty energy saving (thrifty) which implies that the temperature setpoint change is set equal to the lower limit ($\Delta T_{sp} = \Delta T_{low}$).

Figure 6.3 contains the obtained simulation results: the normalized operational cost of the smart and thrifty strategies are always less than the operational cost of the zero temperature setpoint change strategy. Moreover, after sometime the operational cost of the smart strategy is always less than the operational cost of the thrifty strategy. Whereas the temperature deviation of the smart strategy is closer to zero which means that the customer comfort is less compromised.

In the thesis it is also demonstrated that the temperature setpoint changes boundaries (ΔT_{low} and ΔT_{low}) define the payment reduction, which can be up to 20% at $\Delta T_{low} = -4$°C, $\Delta T_{up} = 4$°C. Moreover, it is demonstrated that the optimization algorithm can be run in real time.

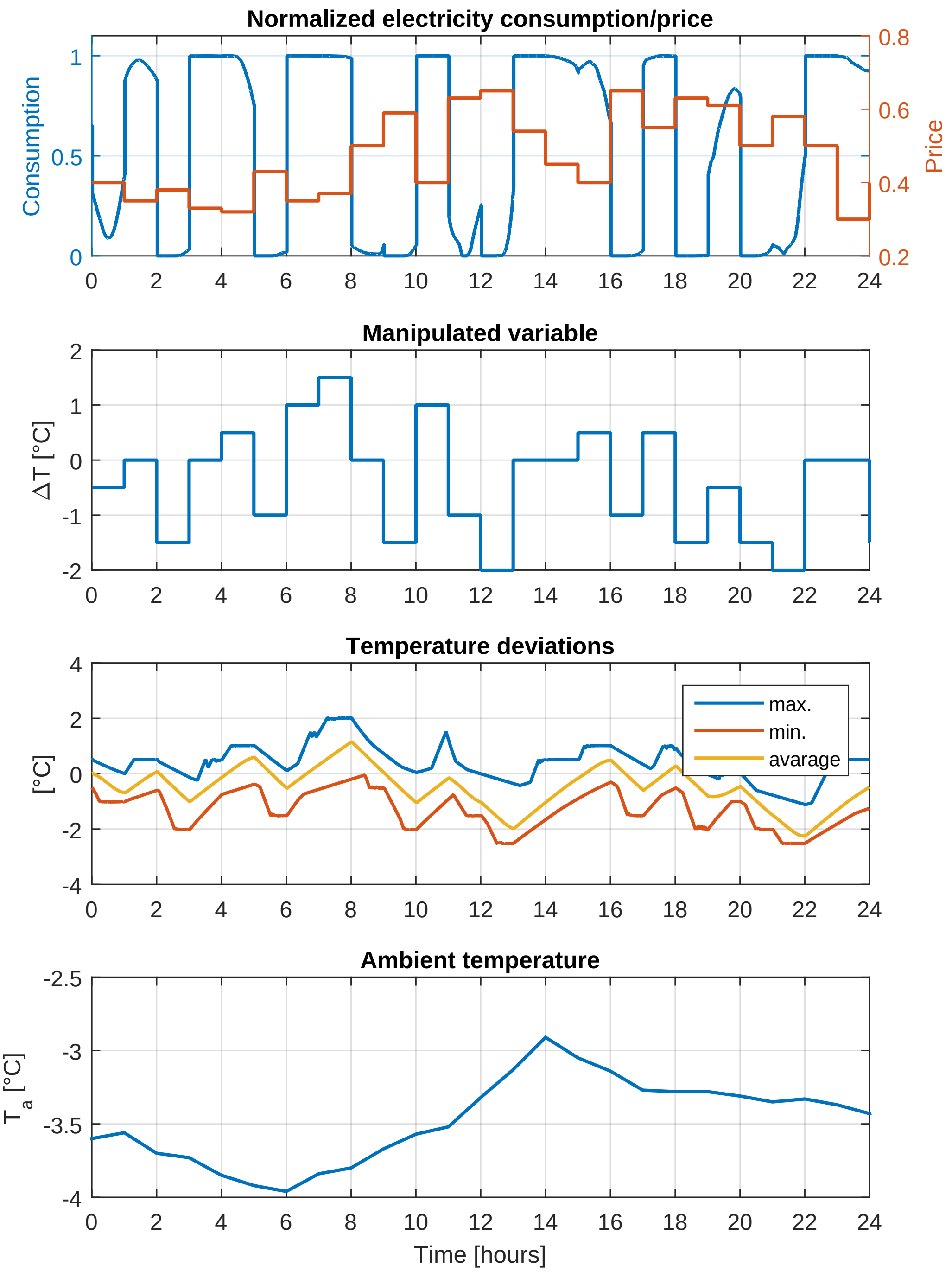

Figure 6.2: Simulation results: performance test

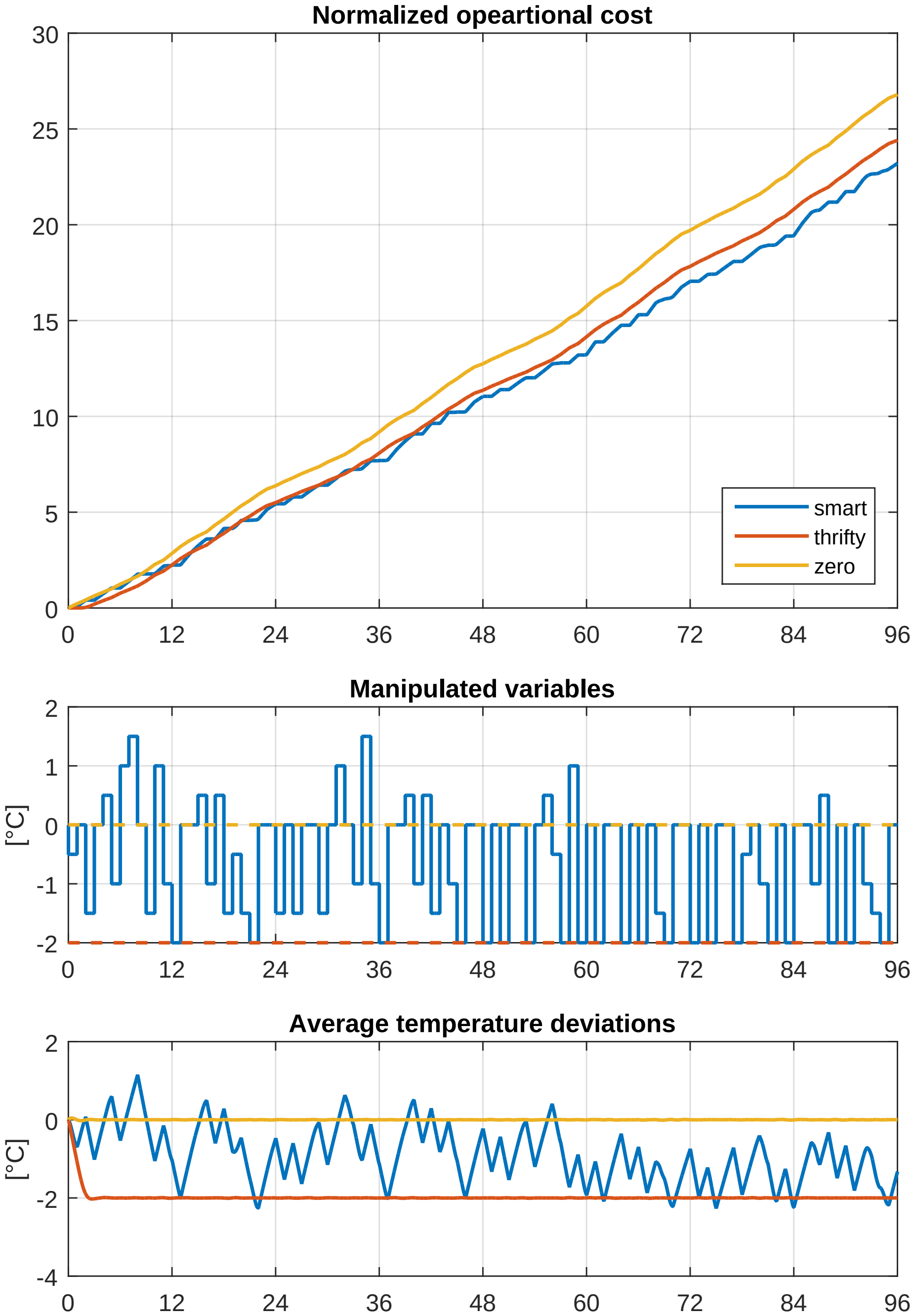

Figure 6.3: Comparison of the energy saving strategies: $\Delta T_{low} = -2,\ \Delta T_{up} = 2$

The summary

This thesis presents a study about utilizing high potential of Thermostatically Controlled Loads (TCLs) for providing regulation reserve in Smart Energy Grids using advanced control techniques. There are two main questions addressed in the work. First, developing a Model Predictive Control (MPC) based on a Linear Parameter-Varying (LPV) model for optimizing energy consumption of a system with TCL. Second, indirect control of a large population of systems with TCLs in energy grid with variable electricity price. According to the concept of energy system with variable electricity price, the proposed economic control strategies are assumed to be applied on the customer side, so-called price responsive consumers. The main objective of the control strategies is to minimize operational cost of either a single unit or the whole population, taking into account forecasted electricity price change which consequently stabilizes the energy system.

The first part of the thesis presents the detailed description of the Economic Model Predictive Control (E-MPC) based on an LPV model. The method requires the variables influencing the model parameters to be known for prediction horizon at each sampling time. Thus, it is able to predict the influence of the parameters variation on the system dynamics.

The modified method was verified by developing control system for a swimming pool heating system described by an LPV model. The parameters of the model vary depending on the wind speed and ambient temperature that can be predicted. The simulation results demonstrate that the presented method can handle the parameters variation and that the energy consumption is shifted to the low-price periods, which corresponds to the economically optimal regime.

The second part of the thesis deals with aggregate control of a relatively large population of TCLs. A new model, based on non-linear modification of bin state transition model, for aggregate demand response approximation is proposed. The modification provides accurate aggregate response of the population of TCLs to

temperature setpoint change signal, which indirectly influences the states of the loads. In the original model, the control strategy generating the switching signal should ensure that the customer comfort is not compromised besides optimizing the aggregate consumption. Whereas, the proposed approach lets the local thermostats to deal with the customers comfort which can be easily managed by limiting the temperature setpoint change, thus reducing the complexity of the control system.

There are several key advantages of the proposed aggregate model that are important for model-based control system design. Firstly, the model is relatively accurate compared to the other models providing aggregate response to the temperature setpoint change. High accuracy is achieved by reusing the idea of original bin state transition model, which implicitly tracks the state of each load in the population. Secondly, the model deals with different levels of heterogeneity of the population, which is verified through simulations. Thirdly, the complexity of the model does not depend on the number of units in the population, because it does not influence the structure or order of the model.

The proposed model was used for formulating the price-responsive control strategy, based on the means of E-MPC, which coordinates the population of TCLs. The controller employs the developed aggregate model for predicting demand response of the population. Taking into account the dynamics of the population allows to better schedule the temperature setpoints for the loads compared to the earlier proposed strategies where the temperature setpoint is defined as a linear function of the electricity price. Since the model is non-linear, the optimization problem is non-linear as well and solved using gradient and adjoint sensitivity analysis methods.

Simulation results demonstrate effectiveness of the proposed control strategy. Firstly, it allows to reduce the operational cost of the whole population up to 20%. The reduction depends on the specified temperature setpoint change limits; in other words, it allows the customer to find a compromise between reducing the electricity payments and their comfort. Secondly, the proposed optimal regime is more efficient than setting the temperature setpoint equal to the lower limit: the operational cost is lower, the customer comfort is less compromised. Thirdly, the computation time analysis demonstrates that the algorithm can run in real time. Moreover, there is a margin that allows to increase the population size and/or run the algorithms on microcontroller units or programmable logical controllers.

References

[1] N. Nakićenović, A. Grübler, and A. McDonald, *Global energy perspectives.* Cambridge University Press, 1998.

[2] I. Dincer, "Renewable energy and sustainable development: a crucial review," *Renewable and Sustainable Energy Reviews*, vol. 4, no. 2, pp. 157–175, 2000.

[3] T. Boden, G. Marland, and R. Andres, "Global, regional, and national co2 emissions," 2009.

[4] "Energy," https://europa.eu/european-union/topics/energy_en, accessed: 2018-02-18.

[5] F. P. Sioshansi, *Smart grid: integrating renewable, distributed and efficient energy.* Academic Press, 2011.

[6] P. Landsbergen, "Feasibility, beneficiality, and institutional compatibility of a micro-chp virtual power plant in the netherlands," 2009.

[7] D. Pudjianto, C. Ramsay, and G. Strbac, "Virtual power plant and system integration of distributed energy resources," *IET Renewable Power Generation*, vol. 1, no. 1, pp. 10–16, 2007.

[8] Z. Stepančič, A. Krpič, A. Kos, K. Koželj, D. Bobek, A. Savanović, and D. Gabrijelčič, "Prosumer flexibility management in smart grids," in *2018 41st International Convention on Information and Communication Technology, Electronics and Microelectronics (MIPRO)*, May 2018, pp. 0432–0437.

[9] D. S. Callaway and I. A. Hiskens, "Achieving controllability of electric loads," *Proceedings of the IEEE*, vol. 99, no. 1, pp. 184–199, 2011.

[10] K. Schisler, T. Sick, and K. Brief, "The role of demand response in ancillary services markets," in *Transmission and Distribution Conference and Exposition, 2008. T&D. IEEE/PES.* IEEE, 2008, pp. 1–3.

[11] N. Lu, "An evaluation of the hvac load potential for providing load balancing service," *IEEE Transactions on Smart Grid*, vol. 3, no. 3, pp. 1263–1270, 2012.

[12] F. L. Alvarado, "Controlling power systems with price signals," *Decision Support Systems*, vol. 40, no. 3-4, pp. 495–504, 2005.

[13] S. P. Holland and E. T. Mansur, "The short-run effects of time-varying prices in competitive electricity markets," *The Energy Journal*, pp. 127–155, 2006.

[14] A. Jokić, M. Lazar, and P. P. van den Bosch, "Real-time control of power systems using nodal prices," *International Journal of Electrical Power & Energy Systems*, vol. 31, no. 9, pp. 522–530, 2009.

[15] A. J. Roscoe and G. Ault, "Supporting high penetrations of renewable generation via implementation of real-time electricity pricing and demand response," *IET Renewable Power Generation*, vol. 4, no. 4, pp. 369–382, 2010.

[16] P. Meibom, K. B. Hilger, H. Madsen, and D. Vinther, "Energy comes together in denmark: The key to a future fossil-free danish power system," *IEEE power and energy magazine*, vol. 11, no. 5, pp. 46–55, 2013.

[17] O. Corradi, H. Ochsenfeld, H. Madsen, and P. Pinson, "Controlling electricity consumption by forecasting its response to varying prices," *IEEE Transactions on Power Systems*, vol. 28, no. 1, pp. 421–429, 2013.

[18] R. Halvgaard, N. K. Poulsen, H. Madsen, and J. B. Jørgensen, "Economic model predictive control for building climate control in a smart grid," in *Innovative Smart Grid Technologies (ISGT), 2012 IEEE PES.* IEEE, 2012, pp. 1–6.

[19] T. G. Hovgaard, L. F. Larsen, and J. B. Jørgensen, "Flexible and cost efficient power consumption using economic mpc a supermarket refrigeration benchmark," in *Decision and Control and European Control Conference (CDC-ECC), 2011 50th IEEE Conference on.* IEEE, 2011, pp. 848–854.

[20] S. Wang and X. Jin, "Model-based optimal control of vav air-conditioning system using genetic algorithm," *Building and Environment*, vol. 35, no. 6, pp. 471–487, 2000.

[21] Y. Ma, A. Kelman, A. Daly, and F. Borrelli, "Predictive control for energy efficient buildings with thermal storage: Modeling, stimulation, and experiments," *IEEE Control Systems*, vol. 32, no. 1, pp. 44–64, 2012.

[22] M. Maasoumy, B. M. Sanandaji, A. Sangiovanni-Vincentelli, and K. Poolla, "Model predictive control of regulation services from commercial buildings to the smart grid," in *American Control Conference (ACC), 2014*. IEEE, 2014, pp. 2226–2233.

[23] Y. Ma, F. Borrelli, B. Hencey, B. Coffey, S. Bengea, and P. Haves, "Model predictive control for the operation of building cooling systems," *IEEE Transactions on control systems technology*, vol. 20, no. 3, pp. 796–803, 2012.

[24] P. Tatjewski, *Advanced control of industrial processes: structures and algorithms*. Springer Science & Business Media, 2007.

[25] J. M. Maciejowski, *Predictive control: with constraints*. Pearson education, 2002.

[26] J. B. Jorgensen, "Adjoint sensitivity results for predictive control, state-and parameter-estimation with nonlinear models," in *Control Conference (ECC), 2007 European*. IEEE, 2007, pp. 3649–3656.

[27] L. N. Petersen, J. B. Jorgensen, and J. B. Rawlings, "Economic optimization of spray dryer operation using nonlinear model predictive control with state estimation," *IFAC-PapersOnLine*, vol. 48, no. 8, pp. 507–513, 2015.

[28] R. Halvgaard, J. B. Jørgensen, N. K. Poulsen, and H. Madsen, "Model predictive control for smart energy systems," 2014.

[29] W. Zhang, J. Lian, C.-Y. Chang, and K. Kalsi, "Aggregated modeling and control of air conditioning loads for demand response," *IEEE transactions on power systems*, vol. 28, no. 4, pp. 4655–4664, 2013.

[30] G. K. Larsen, N. D. van Foreest, and J. M. Scherpen, "Distributed control of the power supply-demand balance," *IEEE Transactions on Smart Grid*, vol. 4, no. 2, pp. 828–836, 2013.

[31] S. Rahnama, J. Stoustrup, and H. Rasmussen, "Model predictive control for integration of industrial consumers to the smart grid under a direct control policy," in *Control Applications (CCA), 2013 IEEE International Conference on*. IEEE, 2013, pp. 515–520.

[32] C. Perfumo, J. H. Braslavsky, and J. K. Ward, "Model-based estimation of energy savings in load control events for thermostatically controlled loads," *IEEE Transactions on Smart Grid*, vol. 5, no. 3, pp. 1410–1420, 2014.

[33] J. L. Mathieu, S. Koch, and D. S. Callaway, "State estimation and control of electric loads to manage real-time energy imbalance," *IEEE Transactions on Power Systems*, vol. 28, no. 1, pp. 430–440, 2013.

[34] S. Bashash and H. K. Fathy, "Modeling and control of aggregate air conditioning loads for robust renewable power management," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 4, pp. 1318–1327, 2013.

[35] Y. Lin, P. Barooah, and S. P. Meyn, "Low-frequency power-grid ancillary services from commercial building hvac systems," in *Smart Grid Communications (SmartGridComm), 2013 IEEE International Conference on*. IEEE, 2013, pp. 169–174.

[36] F. Oldewurtel, D. Sturzenegger, G. Andersson, M. Morari, and R. S. Smith, "Towards a standardized building assessment for demand response," in *Decision and Control (CDC), 2013 IEEE 52nd Annual Conference on*. IEEE, 2013, pp. 7083–7088.

[37] S. P. Meyn, P. Barooah, A. Bušić, Y. Chen, and J. Ehren, "Ancillary service to the grid using intelligent deferrable loads," *IEEE Transactions on Automatic Control*, vol. 60, no. 11, pp. 2847–2862, 2015.

[38] H. Hao, B. M. Sanandaji, K. Poolla, and T. L. Vincent, "Aggregate flexibility of thermostatically controlled loads," *IEEE Transactions on Power Systems*, vol. 30, no. 1, pp. 189–198, 2015.

[39] W. Zhang, K. Kalsi, J. Fuller, M. Elizondo, and D. Chassin, "Aggregate model for heterogeneous thermostatically controlled loads with demand response," in *Power and Energy Society General Meeting, 2012 IEEE*. IEEE, 2012, pp. 1–8.

[40] H. Hao, B. M. Sanandaji, K. Poolla, and T. L. Vincent, "Potentials and economics of residential thermal loads providing regulation reserve," *Energy Policy*, vol. 79, pp. 115–126, 2015.

[41] A. Noureddine, A. Alouani, and S. Patil, "On the probability density function of an air conditioner duty cycle," in *American Control Conference, 1991*. IEEE, 1991, pp. 2089–2093.

[42] L. C. Totu, "Large scale demand response of thermostatic loads," Ph.D. dissertation, Ph. D. dissertation, Faculty of Engineering and Science, Aalborg University, 2015.

[43] B. M. Sanandaji, H. Hao, and K. Poolla, "Fast regulation service provision via aggregation of thermostatically controlled loads," in *System Sciences (HICSS), 2014 47th Hawaii International Conference on*. IEEE, 2014, pp. 2388–2397.

[44] S. Koch, J. L. Mathieu, and D. S. Callaway, "Modeling and control of aggregated heterogeneous thermostatically controlled loads for ancillary services," in *Proc. PSCC*, 2011, pp. 1–7.

[45] J. Czarnecki and CSIRO, *Swimming Pool Heating by Solar Energy*, ser. CSIRO Division of Mechanical Engineering. Technical report no. TR 19. CSIRO Division of Mechanical Engineering, 1978.

[46] S.-E. Pohl, "Use of air conditioning heat rejection for swimming pool heating," Ph.D. dissertation, University of Wisconsin–Madison, 1999.

[47] "Coefficient of performance," http://industrialheatpumps.nl/en/how_it_works/cop_heat_pump/, accessed: 2018-08-25.

[48] "Smartnet project," http://smartnet-project.eu/the-project/, accessed: 2018-07-28.

[49] "Citites - centre for it–intelligent energy systems in cities," http://smart-cities-centre.org/, accessed: 2018-07-28.

[50] "How low can nordic heat pumps go?" https://www.nordicghp.com/2017/01/heat-pump-effective-temperature-range/, accessed: 2018-10-28.

[51] N. Zemtsov, J. Hlava, G. Frantsuzova, H. Madsen, R. G. Junker, and J. B. Jørgensen, "Economic mpc based on lpv model for thermostatically controlled loads," in *Control and Communications (SIBCON), 2017 International Siberian Conference on*. IEEE, 2017, pp. 1–5.

[52] J. B. Jørgensen, J. K. Huusom, and J. B. Rawlings, "Finite horizon mpc for systems in innovation form," in *Decision and Control and European Control Conference (CDC-ECC), 2011 50th IEEE Conference on*. IEEE, 2011, pp. 1896–1903.

[53] C. N. Perfumo *et al.*, "Dynamic modelling and control of heterogeneous populations of thermostatically controlled loads," 2013.

[54] J. Antonín, "Průzkum fondu budov a možností úspor energie," *Rešerše stávajících studií a vỳpočtové ověření pro rezidenční budovy. Šance pro budovy, prosinec*, 2013.

[55] S. Bashash and H. K. Fathy, "Modeling and control insights into demand-side energy management through setpoint control of thermostatic loads," in *American Control Conference (ACC), 2011*. IEEE, 2011, pp. 4546–4553.

[56] N. Zemtsov, J. Hlava, G. Frantsuzova, H. Madsen, and J. B. Jørgensen, "Economic nonlinear mpc for a population of thermostatically controlled loads," *Computer Science-Research and Development*, vol. 33, no. 1-2, pp. 157–167, 2018.

[57] S. Lloyd, "Least squares quantization in pcm," *IEEE transactions on information theory*, vol. 28, no. 2, pp. 129–137, 1982.

[58] L. N. Petersen and J. B. Jorgensen, "Real-time economic optimization for a fermentation process using model predictive control," in *Control Conference (ECC), 2014 European*. IEEE, 2014, pp. 1831–1836.

[59] L. N. Petersen, N. K. Poulsen, H. H. Niemann, C. Utzen, and J. B. Jørgensen, "Economic optimization of spray dryer operation using nonlinear model predictive control," in *Decision and Control (CDC), 2014 IEEE 53rd Annual Conference on*. IEEE, 2014, pp. 6794–6800.

Curriculum Vitae

Nikita Zemtsov was born in Novosibirsk, Russian Federation, in 1992. He received his bachelor degree in Automation and Control at Novosibirsk State Technical University (NSTU) in 2012. During his master degree in Electrical Engineering and Informatics he was participating in double degree program at NSTU and Technical University of Liberec (TUL), Czech Republic. As a part of this program he also spent 1 semester at Zittau/Görlitz University, Germany, as an ERASMUS student. He received the master degree from both TUL and NSTU in 2014. In the same year he started double degree Ph.D. program at the same universities in Electrical Engineering and Informatics. The main focus of the studies was on predictive control in smart grids.

Nikita is author or co-author of more than 13 conference papers and journal articles in both Russian and English languages. Most of them are related to his Ph.D. studies and were presented at various international conferences. The final results were published in the Computer Science - Research and Development Springer journal and presented at the D-A-CH+ Energieinformatik 2017 conference in Lugano, Switzerland.

As a part of his Ph.D. program he attended an internship at Technical University of Denmark (DTU), where he worked for almost two semesters as a research assistant in 2016/17 academic year. During the internship he was developing Economic Model Predictive Controller for swimming pool heating system applying the results of his research. This work was done within the frameworks of the SmartNet and CITIES projects.

List of Publications

[1] Jaroslav Hlava and Nikita Zemtsov. Aggregated control of electrical heaters for ancillary services provision. In *System Theory, Control and Computing (ICSTCC), 2015 19th International Conference on*, pages 508–513. IEEE, 2015.

[2] Jaroslav Hlava and Nikita Zemtsov. Secondary control with thermostatically controlled loads using mpc based on extended bin state transition model. In *Methods and Models in Automation and Robotics (MMAR), 2016 21st International Conference on*, pages 619–624. IEEE, 2016.

[3] Jaroslav Hlava, Nikita Zemtsov, and Galina Frantsuzova. Application of pid controller based on the localization method for ancillary service provision. In *Control and Communications (SIBCON), 2016 International Siberian Conference on*, pages 1–6. IEEE, 2016.

[4] Nikita Zemtsov and Galina Frantsuzova. Using thermostatically controlled loads for the secondary control provision based on the modal method. In *Actual Problems of Electronics Instrument Engineering (APEIE), 2016 13th International Scientific-Technical Conference on*, volume 3, pages 135–139. IEEE, 2016.

[5] Nikita Zemtsov and Jaroslav Hlava. Direct control of a large population of electrical space heaters based on bin state transition model. In *Control and Automation (MED), 2016 24th Mediterranean Conference on*, pages 310–315. IEEE, 2016.

[6] Nikita Zemtsov, Jaroslav Hlava, and Galina Frantsuzova. Using the robust pid controller to manage the population of thermostatically controlled loads. In *Industrial Engineering, Applications and Manufacturing (ICIEAM), 2017 International Conference on*, pages 1–4. IEEE, 2017.

[7] Nikita Zemtsov, Jaroslav Hlava, Galina Frantsuzova, Henrik Madsen, and John Bagterp Jørgensen. Economic nonlinear mpc for a population of ther-

mostatically controlled loads. *Computer Science-Research and Development*, 33(1-2):157–167, 2018.

[8] Nikita Zemtsov, Jaroslav Hlava, Galina Frantsuzova, Henrik Madsen, Rune Grønborg Junker, and John Bagterp Jørgensen. Economic mpc based on lpv model for thermostatically controlled loads. In *Control and Communications (SIBCON), 2017 International Siberian Conference on*, pages 1–5. IEEE, 2017.